

SEQUENCES AND SERIES

- 1 The third and fourth terms of a geometric series are 27 and $20\frac{1}{4}$ respectively.
 - a Find the first term of the series.
 - b Find the sum to infinity of the series.

- 2 The first three terms of a geometric series are $(k - 8)$, $(k + 4)$ and $(3k + 2)$ respectively, where k is a positive constant.
 - a Find the value of k .
 - b Find the sixth term of the series.
 - c Show that the sum of the first ten terms of the series is 50 857.3 to 1 decimal place.

- 3 The second and fifth terms of a geometric series are 75 and 129.6 respectively.
 - a Show that the first term of the series is 62.5
 - b Find the value of the tenth term of the series to 1 decimal place.
 - c Find the sum of the first 12 terms of the series to 1 decimal place.

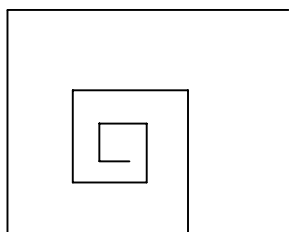
- 4 a Prove that the sum, S_n , of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$
 - b A geometric series has first term 2 and common ratio $\sqrt{2}$.
Given that the sum of the first n terms of the series is $126(\sqrt{2} + 1)$, find the value of n .

- 5 The first term of a geometric series is 18 and the sum to infinity of the series is 15.
 - a Find the common ratio of the series.
 - b Find the third term of the series.
 - c Find the exact difference between the sum of the first eight terms of the series and the sum to infinity of the series.

- 6 The sum of the first n terms of a geometric series is given by $5(3^n - 1)$.
 - a Show that the third term of the series is 90.
 - b Find an expression for the n th term of the series in the form $k(3^n)$ where k is an exact fraction.

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A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous one and at right angles to it. The first line is 4 mm long and thereafter each line is 25% longer than the previous one, so that a spiral is formed as shown above.

- a Find the length, in mm, of the eighth straight line drawn by the program.
- b Find the total length of the spiral, in metres, when 20 straight lines have been drawn.

- 8 The second and fourth terms of a geometric series are 30 and 2.7 respectively.
Given that the common ratio, r , of the series is positive,
- find the value of r and the first term of the series,
 - find the sum to infinity of the series.
- 9 a Evaluate $\sum_{r=3}^{10} 3^r$.
- b Show that $\sum_{r=1}^{15} (2^r - 12r) = 64\,094$.
- 10 A geometric series has common ratio r and the n th term of the series is denoted by u_n .
Given that $u_1 = 64$ and that $u_3 - u_2 = 20$,
- show that $16r^2 - 16r - 5 = 0$,
 - find the two possible values of r ,
 - find the fourth term of the series corresponding to each possible value of r .
 - Taking the value of r such that the series converges, find the sum to infinity of the series.
- 11 A geometric series has first term 4 and common ratio $\frac{1}{2}$.
- Find the eighth term of the series as an exact fraction.
 - Find the n th term of the series in the form 2^y where y is a function of n .
 - Show that the sum of the first n terms of the series is $8 - 2^{3-n}$.
- 12 The sequence of terms u_1, u_2, u_3, \dots is defined by
- $$u_n = 4 \times 3^n, \quad n \geq 1.$$
- Find u_6 .
 - Find the smallest value of t such that the sum of the first t terms of the sequence is greater than 10^{25} .
- 13 The sum of the first and third terms of a geometric series is 150. The sum of the second and fourth terms of the series is -75 .
- Find the first term and common ratio of the series.
 - Find the sum to infinity of the series.
- 14 Three consecutive terms of an arithmetic series are a, b and $(3a + 4)$ respectively.
- Find an expression for b in terms of a .
- Given also that a, b and $(6a + 1)$ respectively are consecutive terms of a geometric series and that a and b are integers,
- find the values of a and b .
- 15 When a ball is dropped onto a horizontal floor it bounces such that it reaches a maximum height of 60% of the height from which it was dropped.
- Find the maximum height the ball reaches after its fourth bounce when it is initially dropped from 3 metres above the floor.
 - Show that when the ball is dropped from a height of h metres above the floor it travels a total distance of $4h$ metres before coming to rest.